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Technical Note

Thermally stratified swirling flows in cylindrical container with co-rotating disks and differentially rotating sidewall

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1. Introduction

Confined swirling flows with heat transfer have relevance to a variety of technological applications [1]. Among these applications, thermally stratified rotating flows are of particular interest, because of their peculiar behaviors. In the previous studies, theoretical [2–4] as well as experimental [5] analyses have been performed in order to describe the feature of rotating stratified flows. Numerical studies have been attempted on the confined swirling flows with vertical temperature difference to illustrate the effect of buoyancy on the secondary flow and heat transfer [6,7]. Results from previous studies indicate that rotating stratified flows respond to the variation of control parameter in a manner consistent with the theoretical predictions. In a most conspicuous case, the bulk fluid approaches to the state of rest under the imposition of large vertical stable temperature difference, regardless of the rotation of end disks [7]. However, above results are obtained for flows in cylindrical containers with sidewall at rest. It should be reminded that the boundary conditions specified at radially outward sidewall exert not a trivial influence on the flows under inhibition of vertical motion by the action of buoyancy. Theoretical studies described only general observations and no attempt was made to solve particular problems. In light of recently invented particle segregation process that utilizes the rotating stratified fluid to separate grains of different sizes [8], the analysis of rotating stratified fluid is of value to the industrial applications. In the present study therefore, stratified swirling flows are numerically solved for a simplified cylindrical container with rotating sidewall. Flow behavior is compared for a fluid with negligible buoyancy and a stratified fluid. The question of whether the bulk of fluid rotates or not under the imposition of large vertical temperature difference is addressed and the influence of sidewall boundary conditions is plainly illustrated.

2. Numerical model

The Stokes streamfunction ψ, the azimuthal vorticity component ω, the azimuthal velocity component v and the temperature T are used as unknown variables under the assumption of axisymmetry. The Boussinesq approximation is also assumed. Governing equations are spatially discretized by the second-order central finite difference schemes and temporary integrated by the first-order explicit Euler method (see [6,9] for details). After all the variables are non-dimensionalized, the Reynolds number Re, the Prandtl number Pr, the Richardson number Ri, the rotation ratio (ratio of the angular velocity of the sidewall to end disks) s and the cylinder aspect ratio h are adopted as governing parameters. The boundary conditions and the cylindrical container are schematically illustrated in Fig. 1. Top and bottom disks of the cylinder rotate at a constant negative angular velocity ($\Omega = -1$) whereas differential rotation ($\Omega_s = s\Omega$) is specified on the sidewall. Top disk is maintained at higher temperature ($T_0 + \Delta T/2$) than the bottom disk ($T_0 - \Delta T/2$) and the sidewall is thermally insulated. Initial condition of each computation was quiescent fluid with linear temperature distribution. Steady state solution was obtained as a converged solution of the governing equations.

3. Results

Steady flows are obtained for $Re = 400$, $Pr = 1.0$, $0 \leqslant Ri \leqslant 5.0$, $-1.0 \leqslant s \leqslant 1.0$ and $h = 2.0$. When the vertical temperature difference is negligibly small ($Ri \sim 0$),

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Nomenclature

g	gravity acceleration
H	height of the cylindrical container
h	cylinder aspect ratio ($h = H/R$)
$\overline{Nu}$	average Nusselt number $\left(\overline{Nu} = 2h \times \int_0^1 \frac{\partial T}{\partial z}\Big\|_{z=0 \text{ or } z=h} r\,\mathrm{d}r\right)$
Pr	Prandtl number ($Pr = \nu/\kappa$)
R	radius of the cylindrical container
Re	Reynolds number ($Re = \Omega R^2/\nu$)
Ri	Richardson number ($Ri = \alpha g \Delta T/\Omega^2 R$)
r	radial coordinate
s	ratio of the angular velocity of sidewall to end disks ($s = \Omega_s/\Omega$)
T	non-dimensional temperature ($T = (T_d - T_0)/\Delta T$, T_d: dimensional temperature)
T_0	average temperature
v	azimuthal velocity component
z	axial coordinate

Greek symbols

α	thermal expansion coefficient
Γ	circulation ($\Gamma = rv$)
ΔT	temperature difference between top and bottom disks
κ	thermal diffusion coefficient
ν	kinematic viscosity coefficient
φ	azimuthal coordinate
ψ	Stokes streamfunction
Ω	angular velocity of end disks ($\Omega = \Omega_{\text{end disks}}$)
Ω_s	angular velocity of sidewall ($\Omega_s = \Omega_{\text{sidewall}}$)
ω	azimuthal vorticity component

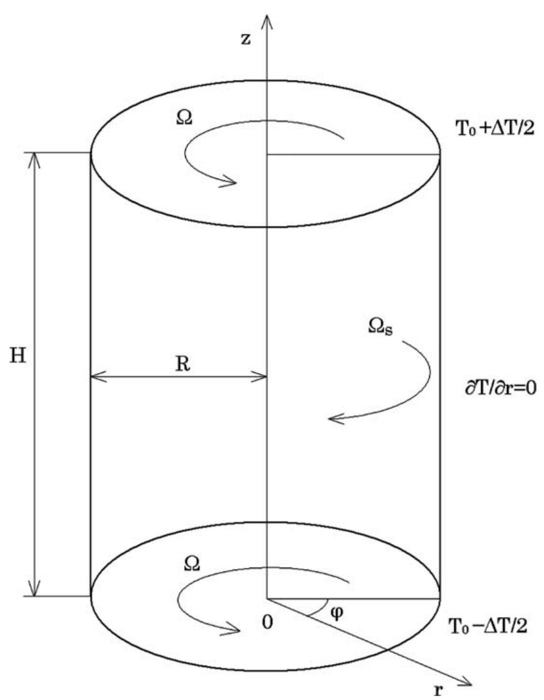

Fig. 1. Schematic illustration of the cylindrical container and the boundary conditions.

the buoyancy term exerts no influence on the momentum transfer. Thus the temperature is passively transferred by the velocity and the flow field is similar to that of homogeneous fluid. On the contrary when ΔT is large ($Ri \gg 0$), stratified fluid emerges.

3.1. Flows under negligible buoyancy (Ri = 0)

The contours are plotted of streamfunction, azimuthal vorticity, azimuthal velocity and isotherms in Fig. 2 for flows with negligible buoyancy ($Ri = 0$) at several selected values of s. When $s = 1.0$, fluid and the container execute rigid rotation as a whole. The fluid motion for $s \sim 1.0$ is explained as a disturbed state from the rigid rotation. Owing to the Ekman suction and the action of induced secondary flows, recirculating zones are created in the vicinity of rotation axis or the mid-height plane at $z = 1.0$ when $s > 0$ (co-rotating case, see the plots for $s = 0.25$ and $s = 0.04$ in Fig. 2). When $s \leqslant 0$ (counter-rotating case) and $s \sim 0$, most of the fluid in the container still rotates in the negative azimuthal direction together with the end disks. Only the fluid in the vicinity of the sidewall rotates in the positive azimuthal direction (thin contour lines in plot (c) for $s = -0.06$ in Fig. 2). As s is gradually decreased below zero, volume of fluid with positive azimuthal velocity component gradually increases when $-0.06 \leqslant s \leqslant 0$. When $s \sim -0.07$, fluid near the mid-height plane (ca. $0.8 \leqslant z \leqslant 1.2$) including fluid in the vicinity of the rotation axis starts to rotate in the positive azimuthal direction. It appears that much of the inner fluid changes the direction of rotation from negative to positive azimuthal direction on a relatively narrow range of rotation ratio s. When s is further decreased ($-1.0 \leqslant s \leqslant -0.08$), bulk of the fluid rotates in positive azimuthal direction with the sidewall and only fluid in the vicinity of the end disks rotates in the negative direction (plots for $s = -1.0$ in Fig. 2).

3.2. Flows of a stratified fluid (Ri = 5.0)

The contour plots of ψ, ω, v and T are shown in Fig. 3 for a thermally stratified fluid case. For all the plots shown in Fig. 3, suppressed Ekman suction is remarked. The volume of fluid that rotates in positive azimuthal direction appears to increase smoothly as s is decreased when

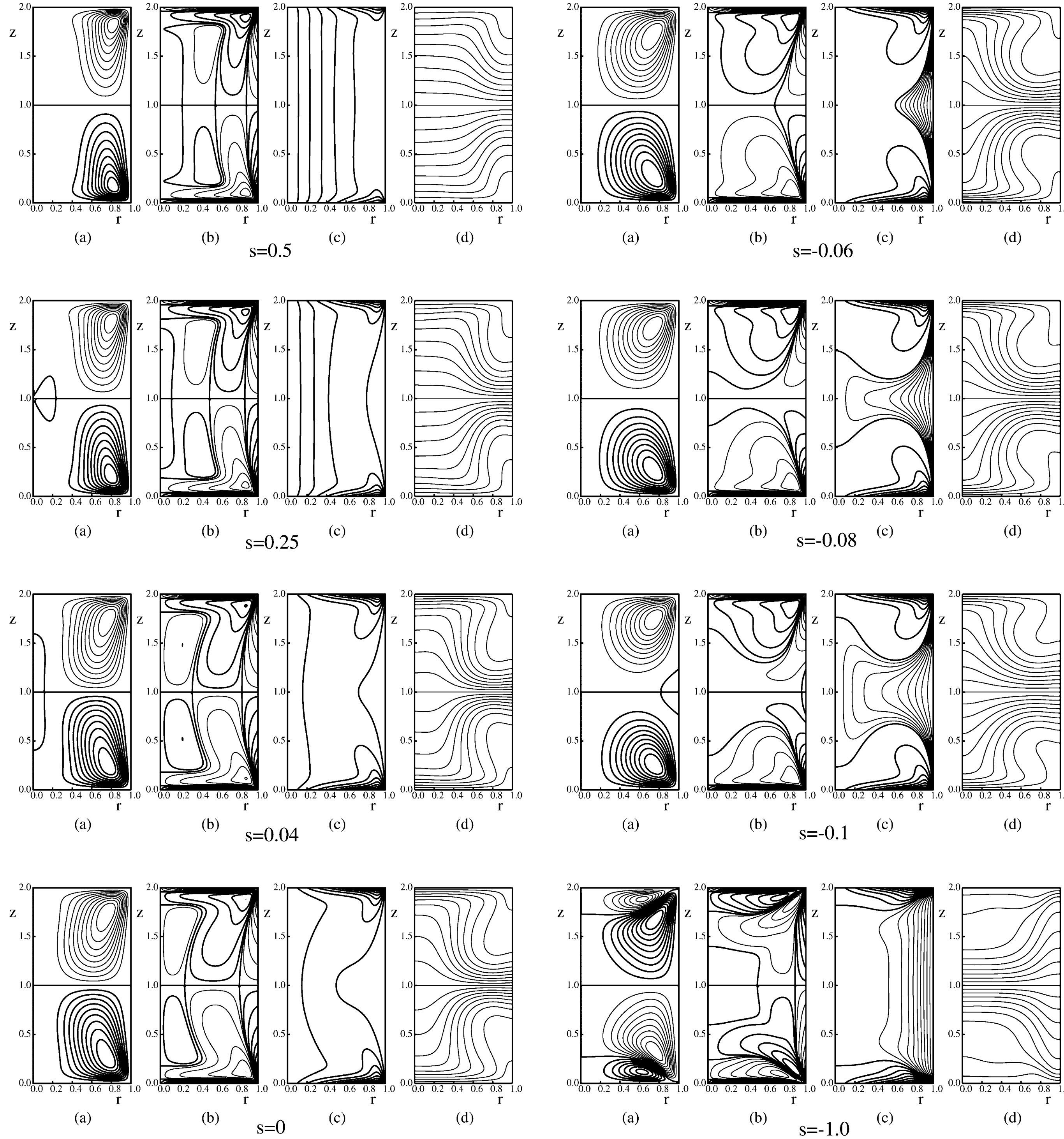

Fig. 2. Contour plots of (a) ψ, (b) ω, (c) v, and (d) isotherms. The values of contour lines are (a) $\psi = \psi_{\max}(i/10)$, $\psi_{\min}(i/10)$, (b) $\omega = \omega_{\max}(i/10)^3$, $\omega = \omega_{\min}(i/10)^3$, (c) $v = -(i/10)$, ($v = s(i/10)$ in case $s < 0$), $i = 0, \ldots, 10$ and (d) $T = (i/20) - (1/2)$, $i = 0, 1, \ldots, 20$, respectively. $Re = 400$, $Ri = 0$.

$s < 0$. Previously observed relatively rapid change in the direction of rotation of the inner fluid as s (<0) is varied is not any more observed for the stratified fluid.

3.3. Discussion

Radial distribution of the azimuthal velocity component at $z = 1.0$ is plotted in Fig. 4(a) for $Ri = 0$. Plots in Fig. 4(a) confirm our observation that when $s > 0$, bulk of the fluid exhibits quasi-rigid rotation ($v \sim -r$). For example, the azimuthal velocity component v for $s = 0$ is approximately proportional to r over $0 \leqslant r \leqslant 0.25$. Plots in Fig. 4(b) display the radial distribution of v for the stratified fluid case. It is observed in these figures that contrary to the previous plots, velocity plots drawn at $z = h/2$ exhibit quasi-linear profile ($v \sim sr$) for all the values of s considered. Radial lin-

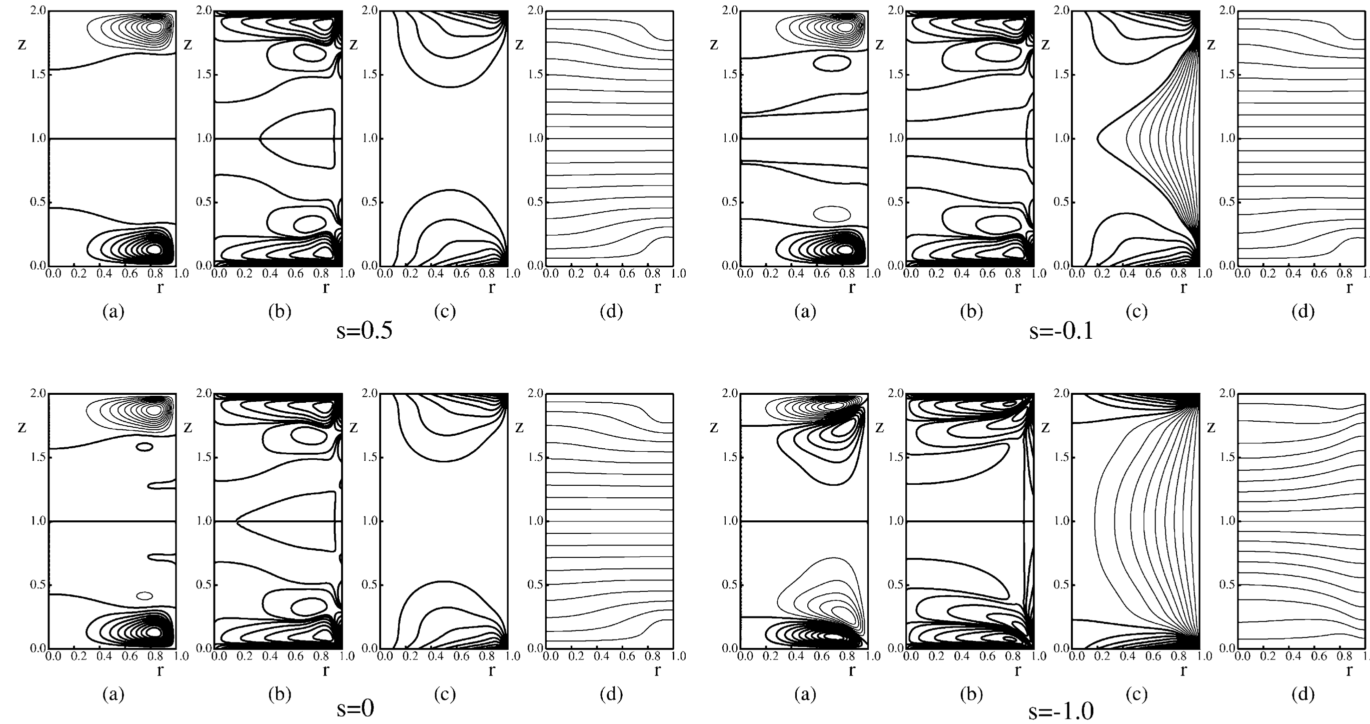

Fig. 3. Similar plots to Fig. 2. $Re = 400$, $Ri = 5.0$.

ear velocity profile indicates that the fluid at this height rotates quasi-rigidly with the sidewall and the boundary layer is not formed on the sidewall. When large vertical temperature difference is imposed on the system, the Ekman pumping of the boundary layer on the disks is weakened. As a result of the loss of secondary flows otherwise induced by the pumping, the azimuthal momentum is not convected from the rotating disks into the inner fluid. Accordingly, the direction of rotation of the bulk fluid is determined by the sidewall boundary conditions.

The value of circulation Γ at a point $(r, z) = (1/2, h/2)$ is plotted next as a function of rotation ratio s in Fig. 4(c). Plots in Fig. 4(c) summarize the behavior of the fluid rotation as s is varied. The value of Γ at $s = 1.0$ is obviously 1/4. It is noted for $Ri = 0$ that the circulation at this point changes its sign between $-0.2 < s < 0$ for the value of Re studied. Contrary to this behavior, for the stratified fluid case ($Ri > 0$), it is readily noticed that the value of Γ is approaching toward $\Gamma \sim -(1/4)s$ as Ri is increased. In accordance with the observations made in the plots of azimuthal velocity component, change of the value of Γ as s is varied is gradual and almost linear with respect to s for the stratified fluid.

In the previous numerical studies, stratified flows are investigated in cylindrical container with rotating top disk [6], or in a cylinder with co-/counter-rotating disks [7]. Stationary sidewall is assumed in all of these studies. Flows under large vertical temperature difference for these boundary conditions exhibit quiescent inner fluid. When the effect of buoyancy exceeds the suction of boundary layers on rotating disks, secondary flows are suppressed and azimuthal momentum is not supplied to the inner fluid from the disks. Thus fluid in the bulk of container tends to be stationary. In the present flow configuration, stratified bulk fluid tends to rotate with the angular velocity of rotating sidewall. It is observed that in the absence of Ekman suction, the azimuthal momentum is conducted from the rotating sidewall into the bulk fluid. As a result of suppressed suction, the bulk fluid tends to be stationary as seen from the rotating framework of the coordinate fixed to the rotating sidewall.

In order to supplement information on the heat transfer, average Nusselt number is shown in Fig. 4(d) as a function of s. Plots of $\overline{Nu}$ exhibit that heat transfer is augmented in the vicinity of $s = -0.05$ when $Ri = 0$. For $Ri \sim 1.0$, maximum value of $\overline{Nu}$ is attained at $s \sim 0.1$. For larger values of Ri, maximum augmentation occurs once again near $s \sim 0$, though the value of $\overline{Nu}$ is decreased toward 1.0. No clear explanation appears to be possible why $\overline{Nu}$ has a peak at $s \fallingdotseq -0.05$ for $Ri = 0$ and this peak shifts toward positive s for $Ri > 0$. However, it is noted that when $\overline{Nu}$ is maximum, a main circulating zone is observed in the meridional upper and lower planes. The value of $\overline{Nu}$ does not attain maximum when the meridional flow contains several recirculating zones. This observation leads to a conjecture that radial velocity shear at the top and bottom disks is strongest when main circulating zones dominate the secondary flow. This results in the creation of large vertical temperature gradients on the disks and the augmentation of heat transfer from the disks. Above behavior of $\overline{Nu}$ appears to suggest that by adding slight co- or counter-rotation to the sidewall, it is able to adjust heat transfer to attain

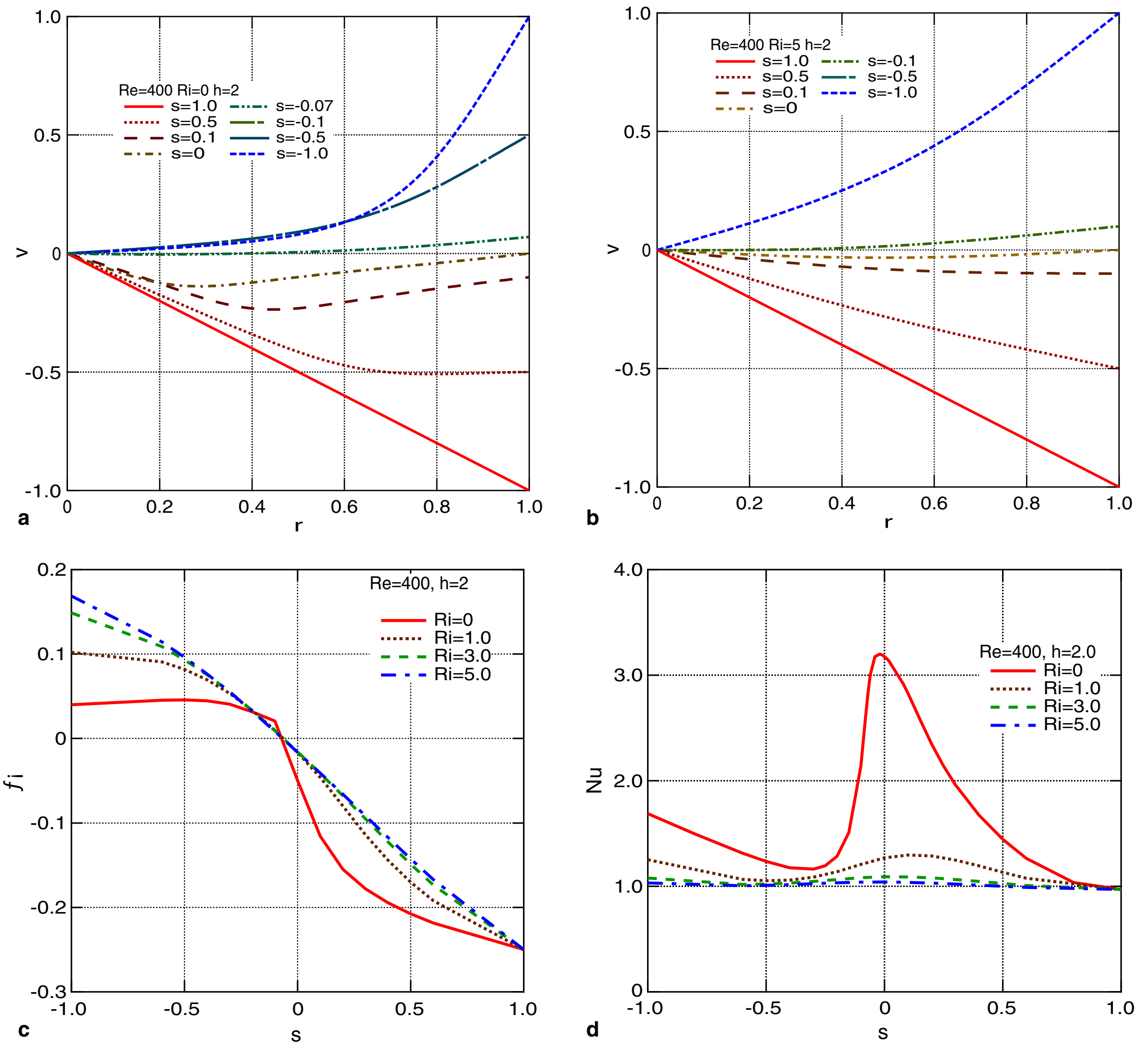

Fig. 4. Distribution of azimuthal velocity component, the value of circulation and the average Nusselt number, $Re = 400$: (a) v at $z = h/2$ vs. r, $Ri = 0$; (b) v at $z = h/2$ vs. r, $Ri = 5.0$; (c) Γ at $(r,z) = (0.5,h/2)$ vs. s, $Ri = 0$, 1.0, 3.0 and 5.0; and (d) $\overline{Nu}$ vs. s, $Ri = 0$, 1.0, 3.0 and 5.0.

maximum augmentation for the confined swirling flows in the present configuration.

4. Conclusion

Numerical study is presented for axisymmetric steady flows in a cylindrical container with co-rotating disks and differentially rotating sidewall. Vertically stable temperature difference is imposed between the top and bottom disks. Under these boundary conditions, it is shown that the inner fluid tends to rotate quasi-rigidly with the sidewall, and the boundary layers are formed in the vicinity of end disks when imposed vertical temperature difference is large. This behavior is in contrast to that of the fluid with negligible buoyancy; for this case, the bulk of fluid changes the direction of rotation over a relatively narrow range of rotation ratio s (<0) when disks and the sidewall are counter-rotating. Result presented in the present study demonstrates the outcome of losing the secondary flow in the confined rotating stratified fluid. This feature of stratified fluid may be useful to control the rotation of inner fluid in industrial applications where quasi-rigidly rotating stratified fluid layers are useful to segregate particles or to mix chemical substances.

References

[1] Y. Jaluria, Fluid flow phenomena in materials processing – the 2000 Freeman scholar lecture, ASME Trans. J. Fluids Eng. 123 (2001) 173–210.
[2] V. Barcilon, J. Pedlosky, Linear theory of rotating stratified fluid motions, J. Fluid Mech. 29 (1967) 1–16.
[3] V. Barcilon, J. Pedlosky, On the steady motions produced by a stable stratification in a rapidly rotating fluid, J. Fluid Mech. 29 (1967) 673–690.
[4] H.P. Greenspan, The Theory of Rotating Fluids, Cambridge University Press (1969) 124–132.
[5] P.F. Linden, The flow of a stratified fluid in a rotating annulus, J. Fluid Mech. 79 (1977) 435–447.

[6] R. Iwatsu, Flow pattern and heat transfer of swirling flows in cylindrical container with rotating top and stable temperature gradient, Int. J. Heat Mass Transfer 47 (2004) 2755–2767.
[7] Y. Omi, R. Iwatsu, Numerical study of swirling flows in a cylindrical container with co-/counter-rotating end disks under stable temperature difference, Int. J. Heat Mass Transfer 48 (2005) 4854–4866.
[8] K. Nakabayashi, Y. Tsuchida, Japan Patent No. H6-150905, 1994.
[9] R. Iwatsu, A second-order axisymmetric flow solver on cylindrical coordinate system, Comput. Fluid Dyn. J. 12 (4) (2004) 595–600.